

The Effects of Collisions with Neutral Particles on the Instability of Two Superposed Composite Plasmas Streaming Through Porous Medium

Mohamed Fahmy El-Sayed

Department of Mathematics and Computer Science, Faculty of Science,
United Arab Emirates University, P. O. Box 17551 Al Ain, U. A. E.

Also at: Department of Mathematics, Faculty of Education,
Ain Shams University, Roxy, Cairo, Egypt

Reprint requests to Dr. M. F. El-Sayed; E-mail: elsayed@nyx.uaeu.ac.ae

Z. Naturforsch. **54 a**, 411–416 (1999); received April 19, 1999

The effects of collisions with neutral atoms on the hydromagnetic stability of the plane interface separating two streaming superposed composite plasmas of uniform densities in a porous medium are investigated. In the absence of fluid velocities, it is found, for a potentially stable configuration, that the system remains stable, while for a potentially unstable configuration, the unstable system becomes stable under a certain condition of the wavenumber depending on the values of the fluid densities, Alfvén velocities, and the orientation of the magnetic field. The porosity of the porous medium does not have any significant effect on the stability criterion. In the presence of fluid velocities, it is found that, the instability criterion is independent of the permeability of the medium and the collision effects with neutral particles. The criterion determining the stability does not depend on the permeability of the medium but depends on the density of neutral particles. The porosity of the medium is found to have a significant effect on both the stability and instability criteria in this case. The role of the permeability of the medium, the collisional frequency, and the porosity of the porous medium on the growth rate of the unstable mode is examined analytically. Routh's test of stability is applied to confirm the above results.

PACS: 47.20.-k; 47.55.Mh; 47.65.+a; 52.30.-q; 52.35.-g.

Key words: Hydrodynamic Stability; Flows through Porous Media; Magnetohydrodynamics; Plasma flow; Instabilities in Plasma.

1. Introduction

The instability of a plane interface separating two uniform superposed streaming fluids under varying assumptions of hydrodynamics and hydromagnetics, has been discussed by Chandrasekhar [1]. For the transverse mode of wave propagation, Hans [2] has studied the effects of collisions with neutral atoms on the Rayleigh-Taylor, and Kelvin-Helmholtz configurations in a composite medium. Several authors, e. g. Bhatia [3], have pointed out that the longitudinal mode of propagation is equally interesting. It is found that the collisions have destabilizing influence on both configurations. Sharma and Srivastava [4], Bhatia and Steiner [5], Callebaut et al. [6], and El-Sayed [7] have studied these stability problems for general perturbations in electro- and magnetohydrodynamics. In all the above studies, the medium has been considered to be non-porous.

The flow through a porous medium has found considerable interest in recent years particularly among geophysical fluid dynamicists [8]. The gross effect, as the fluid slowly percolates through the pores of the rock, is represented by Darcy's law, which describes the flow of an incompressible Newtonian fluid of viscosity μ through a homogeneous and isotropic porous medium of permeability k_1 . Prakash and Manchada [9] studied the Rayleigh-Taylor instability of an infinite, incompressible, homogeneous, conducting fluid in a porous medium in the presence of uniform rotation and suspended particles. On the other hand, the electrohydrodynamic Kelvin-Helmholtz instability problems for the flow in porous medium have been considered by the author [10].

In the present article we study the effect of collisions with neutral particles on the hydromagnetic stability of the plane interface separating two superposed composite plasmas of uniform densities stream-

ing through a porous medium in the presence of a uniform horizontal magnetic field.

2. Formulation and Perturbation Equations

We consider the motion of the mixture of an infinitely conducting, incompressible, and ionized fluid, and a neutral gas through a porous medium, acted on by a magnetic field $\mathbf{H}(H_x, H_y, 0)$, a gravity force $\mathbf{g}(0, 0, -g)$, and a streaming velocity $\mathbf{U}(U, 0, 0)$. We assume that both the conducting fluid and the neutral gas behave like continua, and that the effects on the neutrals resulting from the presence of a magnetic field, and the fields of gravity and pressure can be neglected.

Let $\mathbf{v}(u, v, w)$, $\mathbf{h}(h_x, h_y, h_z)$, $\delta\rho$, and δp denote, respectively, the perturbations in velocity \mathbf{U} , magnetic field \mathbf{H} , density ρ , and pressure p of the conducting fluid, while ρ_d , ν_c , \mathbf{v}_d , and ν denote the density of the neutrals, the collisional (frictional) frequency between the two components of the composite medium, the velocity of the neutral component, and the kinematic viscosity of the conducting fluid, respectively. Then, the linearized perturbation equations governing the motion of the composite medium are

$$\begin{aligned} \frac{\rho}{\varepsilon} \left(\frac{\partial}{\partial t} + \frac{1}{\varepsilon} \mathbf{U} \cdot \nabla \right) \mathbf{v} = & -\nabla \delta p + \frac{\mu_e}{4\pi} (\nabla \times \mathbf{h}) \times \mathbf{H} \\ & + g \delta \rho - \frac{\rho \nu}{k_1} \mathbf{v} \\ & + \frac{\rho_d \nu_c}{\varepsilon} (\mathbf{v}_d - \mathbf{v}), \end{aligned} \quad (1)$$

$$\left(\varepsilon \frac{\partial}{\partial t} + \mathbf{U} \cdot \nabla \right) \mathbf{v}_d = -\varepsilon \nu_c (\mathbf{v}_d - \mathbf{v}), \quad (2)$$

$$\nabla \cdot \mathbf{v} = 0, \quad (3)$$

$$\nabla \cdot \mathbf{h} = 0, \quad (4)$$

$$\left[\varepsilon \frac{\partial}{\partial t} + \mathbf{U} \cdot \nabla \right] \mathbf{h} = \nabla \times (\mathbf{v} \times \mathbf{H}), \quad (5)$$

$$\left[\varepsilon \frac{\partial}{\partial t} + \mathbf{U} \cdot \nabla \right] \delta \rho = -(\mathbf{v} \cdot \nabla) \rho, \quad (6)$$

where μ_e is the magnetic permeability and ε the porosity of the porous medium.

We analyze the disturbances into normal modes by seeking solutions of the above equations whose dependence on x , y , and t of the form

$$\exp(ik_x x + ik_y y + nt) \quad (7)$$

where n is the frequency of the harmonic disturbance,

and k_x , k_y are the horizontal wavenumbers, $k^2 = k_x^2 + k_y^2$.

Eliminating \mathbf{v}_d between (1) and (2), and using (7), the equations (1) - (6) give

$$\begin{aligned} \rho \left[\frac{(\varepsilon n + ik_x U)}{\varepsilon^2} + \frac{\nu}{k_1} + \frac{\beta \nu_c (\varepsilon n + ik_x U)}{\varepsilon (\varepsilon n + ik_x U + \varepsilon \nu_c)} \right] u \\ = -ik_x \delta p + \frac{\mu_e H_y}{4\pi} (ik_y h_x - ik_x h_y), \end{aligned} \quad (8)$$

$$\begin{aligned} \rho \left[\frac{(\varepsilon n + ik_x U)}{\varepsilon^2} + \frac{\nu}{k_1} + \frac{\beta \nu_c (\varepsilon n + ik_x U)}{\varepsilon (\varepsilon n + ik_x U + \varepsilon \nu_c)} \right] v \\ = -ik_y \delta p + \frac{\mu_e H_x}{4\pi} (ik_x h_y - ik_y h_x), \end{aligned} \quad (9)$$

$$\begin{aligned} \rho \left[\frac{(\varepsilon n + ik_x U)}{\varepsilon^2} + \frac{\nu}{k_1} + \frac{\beta \nu_c (\varepsilon n + ik_x U)}{\varepsilon (\varepsilon n + ik_x U + \varepsilon \nu_c)} \right] w \\ = -D \delta p - g \delta \rho + \frac{\mu_e H_x}{4\pi} (ik_x h_z - D h_x) \\ + \frac{\mu_e H_y}{4\pi} (ik_y h_z - D h_y) \end{aligned} \quad (10)$$

$$ik_x u + ik_y v + D w = 0, \quad (11)$$

$$ik_x h_x + ik_y h_y + D h_z = 0, \quad (12)$$

$$(\varepsilon n + ik_x U) \mathbf{h} = (ik_x H_x + ik_y H_y) \mathbf{v}, \quad (13)$$

$$(\varepsilon n + ik_x U) \delta \rho = -w D \rho, \quad (14)$$

where $\beta = \rho_d / \rho$, and $D = d/dz$.

Multiplying (8) and (9) by $-ik_x$ and $-ik_y$, respectively, adding the resulting equations, and using (10) - (14), we obtain

$$\begin{aligned} \left[\frac{(\varepsilon n + ik_x U)}{\varepsilon^2} + \frac{\nu}{k_1} + \frac{\beta \nu_c (\varepsilon n + ik_x U)}{\varepsilon (\varepsilon n + ik_x U + \varepsilon \nu_c)} \right] \\ \cdot \{ D(\rho D w) - k^2 \rho w \} + \frac{\mu_e (k_x H_x + k_y H_y)^2}{4\pi (\varepsilon n + ik_x U)} (D^2 - k^2) w \\ + \frac{g k^2 (D \rho)}{(\varepsilon n + ik_x U)} w = 0. \end{aligned} \quad (15)$$

3. Uniform Composite Media

We consider the case that the two superposed composite media, in which the densities ρ_1 and ρ_2 (and

also ρ_d) are assumed to be uniform, are streaming past each other with uniform streaming velocities U_1 and U_2 , and are separated by a horizontal boundary at $z = 0$. Then, in each region of constant ρ (and the same kinematic viscosity [1]), (15) becomes

$$(D^2 - k^2)w = 0. \quad (16)$$

Since w must be bounded both when $z \rightarrow \infty$ (in the upper fluid), and $z \rightarrow -\infty$ (in the lower fluid), the appropriate solutions of w can be written as

$$w_1 = A(\varepsilon n + ik_x U_1)e^{kz}, \quad z < 0, \quad (17)$$

$$w_2 = A(\varepsilon n + ik_x U_2)e^{-kz}, \quad z > 0, \quad (18)$$

where the same constant A has been chosen in (17) and (18) to ensure the continuity of $w/(\varepsilon n + ik_x U)$ at the interface $z = 0$.

Also, integrating (15) across the interface $z = 0$, we obtain

$$\begin{aligned} \Delta_0 \left\{ \left[\frac{(\varepsilon n + ik_x U)}{\varepsilon^2} + \frac{\nu}{k_1} + \frac{\beta \nu_c (\varepsilon n + ik_x U)}{\varepsilon(\varepsilon n + ik_x U + \varepsilon \nu_c)} \right] \rho D w \right\} \\ + \frac{\mu_e (k_x H_x + k_y H_y)^2}{4\pi} \Delta_0 \left(\frac{D w}{\varepsilon n + ik_x U} \right)_0 \\ + g k^2 \Delta_0(\rho) \left(\frac{w}{\varepsilon n + ik_x U} \right)_0 = 0, \end{aligned} \quad (19)$$

where $\Delta_0(f)$ is the jump that a quantity f experiences at $z = 0$, and $(w/(\varepsilon n + ik_x U))_0$ is the unique value that this quantity has at $z = 0$.

Substituting the values of w_1 and w_2 from (17) and (18) into (19), we obtain the dispersion relation

$$\begin{aligned} \alpha_1(\varepsilon n + ik_x U_1) \\ \cdot \left[\frac{(\varepsilon n + ik_x U_1)}{\varepsilon^2} + \frac{\nu}{k_1} + \frac{\beta_1 \nu_c (\varepsilon n + ik_x U_1)}{\varepsilon(\varepsilon n + ik_x U_1 + \varepsilon \nu_c)} \right] \\ + \alpha_2(\varepsilon n + ik_x U_2) \\ \cdot \left[\frac{(\varepsilon n + ik_x U_2)}{\varepsilon^2} + \frac{\nu}{k_1} + \frac{\beta_2 \nu_c (\varepsilon n + ik_x U_2)}{\varepsilon(\varepsilon n + ik_x U_2 + \varepsilon \nu_c)} \right] \\ + [2(\mathbf{k} \cdot \mathbf{V}_A)^2 - gk(\alpha_2 - \alpha_1)] = 0, \end{aligned} \quad (20)$$

where

$$\alpha_j = \frac{\rho_j}{\rho_1 + \rho_2}, \quad j = 1, 2, \quad \text{and} \quad \mathbf{V}_A = \sqrt{\frac{\mu_e}{4\pi(\rho_1 + \rho_2)}} \mathbf{H}.$$

Equation (20) is similar to the same equation obtained earlier by Sharma et al. [11], except that in their analysis they missed (due to an error in algebra) the parameter ε which indicates the porosity of the porous medium.

4. Stability Analysis and Discussion

Now we shall discuss two cases of interest, i. e. the cases of absence and presence of fluid velocities U_1 and U_2 , respectively.

(i) *Rayleigh-Taylor configuration:*

For the case of no streaming motion (when $U_1 = U_2 = 0$), the dispersion relation (20) reduces to

$$\begin{aligned} n^2(n + \nu_c) + n^2 \left[\frac{\varepsilon \nu}{k_1} + \nu_c(\alpha_1 \beta_1 + \alpha_2 \beta_2) \right] + \frac{\varepsilon \nu \nu_c}{k_1} n \\ + (n + \nu_c) [2(\mathbf{k} \cdot \mathbf{V}_A)^2 - gk(\alpha_2 - \alpha_1)] = 0. \end{aligned} \quad (21)$$

Applying Hurwitz' criterion to the case $\alpha_2 < \alpha_1$ (potentially stable configuration), we find that the medium is stable in the presence of collisions of this mode, the kinematic viscosity of the conducting fluid, and the porosity of the porous medium, also as it is in the absence of them.

For the alternative case when $\alpha_2 > \alpha_1$ (potentially unstable configuration), we find, by applying Hurwitz' criterion again, that the medium is stable for all wavenumbers k such that

$$k > \frac{g(\alpha_2 - \alpha_1)}{(V_1 \cos \theta + V_2 \sin \theta)^2}, \quad (22)$$

where V_1 and V_2 are the Alfvén velocities in the x and y directions, respectively, and $k_x = k \cos \theta$, $k_y = k \sin \theta$, θ being the orientation of the magnetic field. Note that the porosity of the porous medium does not have any effect on the stability criterion in this case.

(ii) *Kelvin-Helmholtz configuration:*

The dispersion relation (20), in its present form, is quite complex. We therefore consider the model used by Hans [2], in which the two media of the same density ($\alpha_1 = \alpha_2 = 1/2$) are flowing across each other with streaming velocities U , $-U$. Thus putting $\beta_1 = \beta_2 = \beta$ in the dispersion relation (20), we obtain

$$\begin{aligned}
& n^4 + n^3 \left[\nu_c(2 + \beta) + \frac{\varepsilon\nu}{k_1} \right] + n^2 \left[\nu_c^2(1 + \beta) + \frac{2\varepsilon\nu\nu_c}{k_1} \right. \\
& \left. + 2(\mathbf{k} \cdot \mathbf{V}_A)^2 \right] + n \left\{ \frac{\varepsilon\nu}{k_1} \left(\nu_c^2 + \frac{k_x^2 U^2}{\varepsilon^2} \right) + \frac{k_x^2 U^2 \beta \nu_c}{\varepsilon^2} \right. \\
& \left. + 2\nu_c \left[2(\mathbf{k} \cdot \mathbf{V}_A)^2 - \frac{k_x^2 U^2}{\varepsilon^2} \right] \right\} + \nu_c^2 \left\{ 2(\mathbf{k} \cdot \mathbf{V}_A)^2 \right. \\
& \left. - \frac{k_x^2 U^2}{\varepsilon^2} (1 + \beta) \right\} + \frac{k_x^2 U^2}{\varepsilon^2} \left\{ 2(\mathbf{k} \cdot \mathbf{V}_A)^2 - \frac{k_x^2 U^2}{\varepsilon^2} \right\} \\
& = 0
\end{aligned} \tag{23}$$

It is evident from (23) that if

$$2(\mathbf{k} \cdot \mathbf{V}_A)^2 > \frac{k_x^2 U^2}{\varepsilon^2} (1 + \beta), \tag{24}$$

there is no change of sign in the quartic equation of n . Equation (23) therefore can not allow any positive root, meaning thereby that the system is stable. If

$$2(\mathbf{k} \cdot \mathbf{V}_A)^2 < \frac{k_x^2 U^2}{\varepsilon^2}, \tag{25}$$

there is one change of sign in (23). Therefore (23) allows one positive root and so the system is unstable. Note that the permeability of the medium k_1 , the collisional frequency ν_c , and the viscosity of the conducting fluid ν , do not appear in the inequalities (24) and (25). The instability criterion (25) is, therefore, independent of the permeability of the medium and the collisional effects with neutral particles as well as the viscosity of the fluid. Also the criterion (24) determining the stability does not depend on either the permeability of the medium nor the fluid viscosity, but depends on the density of the neutral particles. It is clear also that the inequalities (24) and (25) depend on the porosity of the medium.

If the inequality (25) is satisfied, then (23) possesses one positive root implying thereby that the system is unstable. Let n_0 denote the positive root of (23), then we have

$$\begin{aligned}
& n_0^4 + n_0^3 \left[\nu_c(2 + \beta) + \frac{\varepsilon\nu}{k_1} \right] + n_0^2 \left[\nu_c^2(1 + \beta) + \frac{2\varepsilon\nu\nu_c}{k_1} \right. \\
& \left. + 2(\mathbf{k} \cdot \mathbf{V}_A)^2 \right] + n_0 \left\{ \frac{\varepsilon\nu}{k_1} \left(\nu_c^2 + \frac{k_x^2 U^2}{\varepsilon^2} \right) + \frac{k_x^2 U^2 \beta \nu_c}{\varepsilon^2} \right.
\end{aligned}$$

$$\begin{aligned}
& \left. + 2\nu_c \left[2(\mathbf{k} \cdot \mathbf{V}_A)^2 - \frac{k_x^2 U^2}{\varepsilon^2} \right] \right\} + \nu_c^2 \left\{ 2(\mathbf{k} \cdot \mathbf{V}_A)^2 \right. \\
& \left. - \frac{k_x^2 U^2}{\varepsilon^2} (1 + \beta) \right\} + \frac{k_x^2 U^2}{\varepsilon^2} \left\{ 2(\mathbf{k} \cdot \mathbf{V}_A)^2 - \frac{k_x^2 U^2}{\varepsilon^2} \right\} \\
& = 0
\end{aligned} \tag{26}$$

To find the roles of the permeability of the medium, the collisional frequency, and the porosity of the porous medium, on the growth rate of unstable modes, we examine the nature of dn_0/dk_1 , $dn_0/d\nu_c$, and $dn_0/d\varepsilon$, respectively. It follows from (26) that

$$\frac{dn_0}{dk_1} = \frac{\varepsilon\nu n_0}{k_1^2 F} \left[(n_0 + \nu_c)^2 + \frac{k_x^2 U^2}{\varepsilon^2} \right], \tag{27}$$

$$\begin{aligned}
\frac{dn_0}{d\nu_c} &= -\frac{1}{F} \{ n_0^3(2 + \beta) + 2n_0^2[\nu_c(1 + \beta) + (\varepsilon\nu/k_1)] \\
&+ n_0[2\nu_c(\varepsilon\nu/k_1) + 4(\mathbf{k} \cdot \mathbf{V}_A)^2 - (k_x^2 U^2/\varepsilon^2)(2 - \beta)] \\
&+ 2\nu_c[2(\mathbf{k} \cdot \mathbf{V}_A)^2 - (k_x^2 U^2/\varepsilon^2)(1 + \beta)] \},
\end{aligned} \tag{28}$$

$$\begin{aligned}
\frac{dn_0}{d\varepsilon} &= -\frac{1}{\varepsilon F} \{ n_0(\varepsilon\nu/k_1)(n_0 + \nu_c)^2 + n_0(k_x^2 U^2/\varepsilon^2) \\
&\cdot [2\nu_c(2 - \beta) - (\varepsilon\nu/k_1)] + 2(k_x^2 U^2/\varepsilon^2)[\nu_c^2(1 + \beta) \\
&+ 2(k_x^2 U^2/\varepsilon^2) - 2(\mathbf{k} \cdot \mathbf{V}_A)^2] \},
\end{aligned} \tag{29}$$

where

$$\begin{aligned}
F &= 4n_0^3 + 3n_0^2 \left[\nu_c(2 + \beta) + \frac{\varepsilon\nu}{k_1} \right] + 2n_0 \left[\nu_c^2(1 + \beta) \right. \\
&+ 2\nu_c \left(\frac{\varepsilon\nu}{k_1} \right) + 2(\mathbf{k} \cdot \mathbf{V}_A)^2 \left. \right] + \frac{\varepsilon\nu}{k_1} \left[\nu_c^2 + \frac{k_x^2 U^2}{\varepsilon^2} \right] \\
&+ \beta\nu_c \left(\frac{k_x^2 U^2}{\varepsilon^2} \right) + 2\nu_c \left[2(\mathbf{k} \cdot \mathbf{V}_A)^2 - \frac{k_x^2 U^2}{\varepsilon^2} \right].
\end{aligned} \tag{30}$$

It is clear from (27) and (30) that the growth rates may be both increasing or decreasing with the increase in permeability of the medium, as dn_0/dk_1 may be both positive or negative depending on whether the denominator F is positive or negative, respectively. It is also evident from (28) and (29) that the growth rates may both be increasing or decreasing with the increase in both the collisional frequency ν_c and the

porosity of the porous medium ε . Therefore, the permeability of the medium, the collisional frequency, and the porosity of the porous medium have stabilizing as well as destabilizing effect on the considered system depending on dn_0/dk_1 , $dn_0/d\nu_c$, and $dn_0/d\varepsilon$ being positive or negative, respectively.

5. Routh's Criterion for Stability

A necessary condition for all the zeros of a polynomial to be in the left-half plane is that all the coefficients of the polynomial be present and be positive. However, this is not a sufficient condition because under certain circumstances, all the coefficients may be present and positive, and yet the polynomial may have zeros in the right-half plane. Under these conditions, the zeros in the right-half plane will be complex with positive real parts. For the remainder of this section, it is assumed that all the coefficients of the polynomial being considered in (23) are present and are positive.

A general method of writing the polynomial (23) is

$$P(n) = a_4 n^4 + a_3 n^3 + a_2 n^2 + a_1 n + a_0 = 0, \quad (31)$$

where the coefficients $a_4 - a_0$ are given by

$$\begin{aligned} a_4 &= 1, \\ a_3 &= \nu_c(2 + \beta) + \frac{\varepsilon\nu}{k_1}, \\ a_2 &= \nu_c^2(1 + \beta) + \frac{2\varepsilon\nu\nu_c}{k_1} + 2(\mathbf{k} \cdot \mathbf{V}_A)^2, \\ a_1 &= \frac{\varepsilon\nu}{k_1} \left(\nu_c^2 + \frac{k_x^2 U^2}{\varepsilon^2} \right) + \frac{k_x^2 U^2 \beta \nu_c}{\varepsilon^2} \\ &\quad + 2\nu_c \left\{ 2(\mathbf{k} \cdot \mathbf{V}_A)^2 - \frac{k_x^2 U^2}{\varepsilon^2} \right\}, \\ a_0 &= \nu_c^2 \left\{ 2(\mathbf{k} \cdot \mathbf{V}_A)^2 - \frac{k_x^2 U^2}{\varepsilon^2} (1 + \beta) \right\} \\ &\quad + \frac{k_x^2 U^2}{\varepsilon^2} \left\{ 2(\mathbf{k} \cdot \mathbf{V}_A)^2 - \frac{k_x^2 U^2}{\varepsilon^2} \right\}. \end{aligned} \quad (32)$$

The coefficients of the polynomial (31) are placed in two rows as follows [12]:

$$\begin{array}{ccc} a_4 & a_2 & a_0 \\ a_3 & a_1 & \end{array}. \quad (33)$$

The coefficients of the next row are formed from these two rows to be

$$b_2 = \frac{a_3 a_2 - a_4 a_1}{a_3},$$

$$b_0 = \frac{a_3 a_0 - a_4(0)}{a_3},$$

which gives

$$a_3 b_2 = \nu_c^3 (2 + \beta)(1 + \beta) + \frac{\varepsilon\nu\nu_c^2}{k_1} (4 + 3\beta) \quad (34)$$

$$+ \left(\nu_c \beta + \frac{\varepsilon\nu}{k_1} \right) \left\{ 2(\mathbf{k} \cdot \mathbf{V}_A)^2 - \frac{k_x^2 U^2}{\varepsilon^2} \right\} + \frac{2k_x^2 U^2 \nu_c}{\varepsilon^2},$$

$$b_0 = a_0 \quad (35)$$

The zero in the equation for b_0 represents the blank space at the end of the second row in the array of (33).

This row of coefficients is added to the array of (33), resulting in

$$\begin{array}{ccc} a_4 & a_2 & a_0 \\ a_3 & a_1 & \\ b_2 & a_0 & \end{array}. \quad (36)$$

The coefficients of the next row are formed from the last two rows in much the same manner as the row of $b's$ is formed from the first two rows, to get

$$c_1 = \frac{b_2 a_1 - b_3 a_0}{b_2},$$

from which we can write

$$\begin{aligned} a_3 b_2 c_1 &= \left\{ 2(\mathbf{k} \cdot \mathbf{V}_A)^2 - \frac{k_x^2 U^2}{\varepsilon^2} \right\} \{ \nu_c^4 \beta (4 + \beta) \\ &\quad + (2\varepsilon\nu\nu_c^2/k_1)(6 + 5\beta) + (k_x^2 U^2/\varepsilon^2)((\varepsilon\nu\beta/k_1) \\ &\quad - 4\nu_c(1 + \beta)) + (\varepsilon\nu/k_1)(\nu_c^3 \beta - \nu_c(4 + \beta)(k_x^2 U^2/\varepsilon^2)) \} \\ &\quad + \nu_c^2 \beta (k_x^2 U^2/\varepsilon^2) \{ \nu_c(2 + \beta) + (\varepsilon\nu/k_1) \}^2 \\ &\quad + 2\nu_c \left(\nu_c \beta + \frac{\varepsilon\nu}{k_1} \right) \left\{ 2(\mathbf{k} \cdot \mathbf{V}_A)^2 - \frac{k_x^2 U^2}{\varepsilon^2} \right\}^2 \\ &\quad + \frac{\varepsilon\nu\nu_c^5}{k_1} (2 + \beta)(1 + \beta) + \frac{k_x^2 U^2 \nu_c^2}{\varepsilon^2} \left\{ \frac{\varepsilon\nu\nu_c}{k_1} (2 + 7\beta + 4\beta^2) \right. \end{aligned} \quad (37)$$

$$+ \nu_c^2 \beta (2 + \beta) (1 + \beta) + \frac{\varepsilon^2 \nu^2}{k_1^2} (4 + 3\beta) \left\{ \right. \\ \left. + \frac{\varepsilon^2 \nu^2 \nu_c^4}{k_1^2} (4 + 3\beta) \right\}$$

This row of coefficients is added to the array of equation (33), resulting in

$$\begin{array}{ccc} a_4 & a_2 & a_0 \\ a_3 & a_1 & \\ b_2 & a_0 & \\ c_1 & & \end{array} \quad (38)$$

The coefficients of the next row are formed from the last two rows, to be

$$d_0 = \frac{c_1 b_0 - b_2(0)}{c_1} = b_0 = a_0 \quad (39)$$

Therefore, the entire Routh table is completed as

$$\begin{array}{ccc} n^4 : & a_4 & a_2 & a_0 \\ n^3 : & a_3 & a_1 & \\ n^2 : & b_2 & a_0 & \\ n^1 : & c_1 & & \\ n^0 : & a_0 & & \end{array} \quad (40)$$

The column to the left indicated the highest degree in n associated with the appropriate row. The Routh table for a polynomial of any degree is formed in the same manner. Note that a polynomial of degree n has a table that should contain $(n + 1)$ rows. If the polynomial being tested is Hurwitz'ian, there are no

changes of signs in the first column (a_4, a_3, b_2, c_1, a_0). If there are changes in signs in the first column, the polynomial is not Hurwitz'ian. In addition to this, the Routh table yields the information that the number of changes in sign in the first column is equal to the number in zeros of the polynomial in the right-half plane.

The absolute stability of the system can be determined from the location of the zeros of the characteristic polynomial which in turn are the poles of the closed-loop transfer function. If the transfer function has poles in the right-half plane, then the system is unstable, while if all the poles are in the left-half plane, then the system is stable. The preceding analysis has applied Routh's criterion of stability to the characteristic polynomial (23) to determine whether it has any zeros in the right-half plane. From (40) it is clear that there are no changes of signs in the first column (see (32), (34), and (37)) when the condition (24) is satisfied. Therefore, the polynomial of equation (23) has no zeros in the right-half plane, and is therefore Hurwitz'ian. Thus the system is always stable whenever the condition (24) is satisfied, and this result confirms our results in the previous section.

Acknowledgement

The financial support of Natural Sciences and Engineering Research Council of United Arab Emirates University is gratefully acknowledged. I would like also to thank the referee for his useful comments that improved the original manuscript.

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